Sent-Up Examination–2020

Class–XI

Subject : Mathematics

Time : 3 hrs.

Full Marks : 80

Part –B (Marks–10)

1. Choose the correct answer from the given alternatives : 1x10=10

i) The argument of the complex number $Z = 1 + i\tan\frac{3\pi}{5}$ is

a) $\frac{-2\pi}{5}$ (b) $\frac{3\pi}{5}$ (c) $\frac{2\pi}{5}$ (d) $\frac{-3\pi}{5}$

ii) Two variables x and y are related by y=8+2x; if the S.D of x is 3, then the S.D of y will be

a) 10   (b) 14   (c) 11   (d) 6

iii) If the sum of the co-efficients in the expansion of (a+b)ⁿ is 4096, then the greatest coefficient in the expansion is

a) 792   (b) 924   (c) 2924   (d) 1594

iv) If a+b+c = 0, then the straight lines 4ax+3by+c = 0 always pass through a fixed point whose co-irdinates are

a) (4,3) (b) $\left(\frac{1}{4}, \frac{1}{3}\right)'$ (c) $\left(\frac{1}{2}, \frac{1}{3}\right)$ (d) $\left(-\frac{1}{4}, -\frac{1}{3}\right)$

v) The distance between the directrices of the hyperbola x = 8 secθ, y = 8 tanθ is–

a) $16\sqrt{2}$ (b) $4\sqrt{2}$ (c) $8\sqrt{2}$ (d) $6\sqrt{2}$

vi) $\lim\limits_{x \to 0} \dfrac{\sqrt{1-\cos 2x}}{\sqrt{2}x}$ is equal to

    a) 1   (b) –1   (c) 0   (d) limit does not exist.

vii) If the function f be defined by $f(x) = \dfrac{2x+1}{1-3x}$ then $f^{-1}(x)$

    a) $\dfrac{x-1}{3x+2}$ (b) $\dfrac{3x+2}{x-1}$ (c) $\dfrac{1-3x}{2x+1}$ (d) $\dfrac{2x+1}{1-3x}$

viii) In a triangle ABC, if 3a=b+c then the value of $\cot \dfrac{B}{2} \cot \dfrac{C}{2}$ is

    a) 4   (b) 3   (c) 1   (d) 2

ix) The roots of the equation $1-\cos\theta = \sin\theta \sin\dfrac{\theta}{2}$

    a) $\dfrac{n\pi}{4}$ (b) $2n\pi$ (c) $n\pi$ (d) $\dfrac{n\pi}{2}$ where $n \in Z$

x) If $\sin x + \sin^2 x = 1$; then the value of $\cos^{12} x + 3\cos^{10} x + 3\cos^8 x + \cos^6 x$ is–

    a) 1   (b) 4   (c) 2   (d) 3

Group–B

1. a) How many different factors can 2310 have?   2x7=14

<div align="center">OR</div>

If $^nC_{r-1}=36$, $^nC_r = 84$ and $^nC_{r+1} = 126$, find 'n' and 'r'
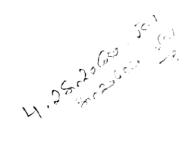
b) Find the middle term or terms in the expassion $\left(2x - \dfrac{1}{3x}\right)^{2n}$

<div align="center">OR</div>

Find the sum of the series 1+3+4+8+7+13+10+18+ ... to 25 terms.

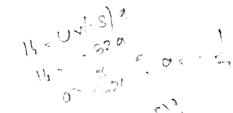c) Find the general solution of $4\sin 4\theta + 1 = \sqrt{5}$

OR

If in a triangle ABC, a=3, b=5 and c=7, show that the triangle is obtuse angled.

d) Prove that $Cos\dfrac{\pi}{5} - Cos\dfrac{2\pi}{5} = \dfrac{1}{2}$

e) Prove that $Cos^4 x + Sin^4 x = \dfrac{1}{2}\left(1 + 2a^2 - a^4\right)$ when a = Sinx+Cosx.

OR

If $Sin2A = \dfrac{4}{5}$ find the vlue of $SinA\left(0 < A < \dfrac{\pi}{4}\right)$.

f) For what value of 'a' will the point (–8, – 4) be an inside point of the parabola $y^2 = 4ax$?

g) Evaluate : $\displaystyle\lim_{h\to 0}\dfrac{e^{\sqrt{x+h}} - e^{\sqrt{x}}}{h}$

OR

$\displaystyle\lim_{h\to 0}\dfrac{Sin(x+h)^2 - Sinx^2}{h}$

Group – C

2. a) If $\dfrac{SinA}{SinC} = \dfrac{Sin(A-B)}{Sin(B-C)}$; then show that a^2, b^2, c^2 are in A.P

OR

In any triangle ABC, If $\dfrac{1}{a+c} + \dfrac{1}{b+c} = \dfrac{3}{a+b+c}$ then show that c = 60°.

b) Find the rank of the word "LATE" when its letters are arranged as in a dictionary.

OR

Find the number of permutations of the letters of the words "FORECAST" and "MILKY" taking 5 at a time of which 3 letters from the first word and 2 from the second.

c) Show that the middle term in the expansion $(1+x)^{2n}$ is

$$\frac{1.3.5\ldots(2n-1).\,2^n.\,x^n}{n!}$$

OR

Prove by induction that $8.7^n + 4^{n+2}$ is divisible by 24 but not by 48 for all $n \in \mathbb{N}$

d) One side of an equilateral triangle is the line 5y=12x–3 and its centroid is at (2,–1) find the length of a side of the triangle.
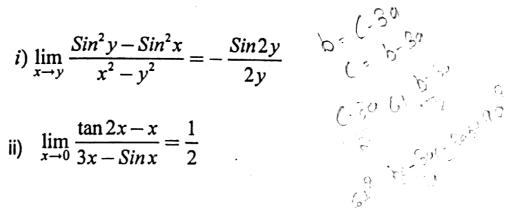
OR

Find the co-ordinates of points on the straight line x+y=4 which are at unit distance from the line 4x+3y = 10.

e) Evaluate

i) $\displaystyle \lim_{x \to y} \frac{Sin^2 y - Sin^2 x}{x^2 - y^2} = -\frac{Sin 2y}{2y}$

ii) $\displaystyle \lim_{x \to 0} \frac{\tan 2x - x}{3x - Sin x} = \frac{1}{2}$

f) Find from the first principle, the desivative of the function w.r.t 'x', (Any one)

   i) $f(x) = e^{\sqrt{\tan x}}$ (ii) $f(x) = \sqrt[3]{\sin x}$

g) If $y = \operatorname{Cosec}\theta + \operatorname{Cot}\theta$; Pove that $\dfrac{2dy}{d\theta} + y^2 + 1 = 0$

OR

Let v and s be the volume and surface respectively of a sphere of radius r. Prove that $\dfrac{2dv}{dt} = \dfrac{r.ds}{dt}$

h) Prove that the following biconditional statement is true :

"The integer 'x' is odd if and only if x^2 is odd"

OR

Given P (A) = a, P (B) = b and $P(A \cap B) = C$. Find the value of the following expressions.

  i) $P(A^c \cup B^c)$ (ii) $P(A^c \cup B)$ (iii) $P(A^c \cap B^c)$ (iv) $P(A \cap B^c)$.

i) The mean and variance of the six values of a variate are 8 and $8\dfrac{2}{3}$. If the four values of the variate be 4,9,11 and 12, find the other two.

OR

For any two events A and B prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Group–D   5x4=20

3. a) Prove that the sum of the reciprocas of the segments of any focal chord

of a parabola is constant.

OR

Find the equation of the ellipse whose foci are (2, 3) and (–2, 3) and

whose semi minor axis is $\sqrt{5}$.

b) If f(x) is differentiable at x=h, find the value of $\lim\limits_{x \to h} \dfrac{(x+h)f(x) - 2hf(x)}{x-h}$

OR

If $\lim\limits_{x \to 0} (1+x)^{1/x} = e$; prove that $\lim\limits_{x \to 0} (1+3x)^{\frac{(x+2)}{x}} = e^6$

c) Find the sum to n-terms of the following series :

$$\left(x+\frac{1}{x}\right)^2 + \left(x^2+\frac{1}{x^2}\right)^2 + \left(x^3+\frac{1}{x^3}\right) + \left(x^4+\frac{1}{x^4}\right)^2 + \dots$$

OR

Find the Co-efficient of term independent of 'x' in the expansion

$$\left(1-\frac{x}{2}+\frac{x^3}{3}\right)\left(x-\frac{1}{x}\right)^7$$

d) If A = 170⁰ ; prove that $\tan\dfrac{A}{2} = \dfrac{-1-\sqrt{1+\tan^2 A}}{\tan A}$

OR

Find the value of $\tan\dfrac{\pi}{32}$